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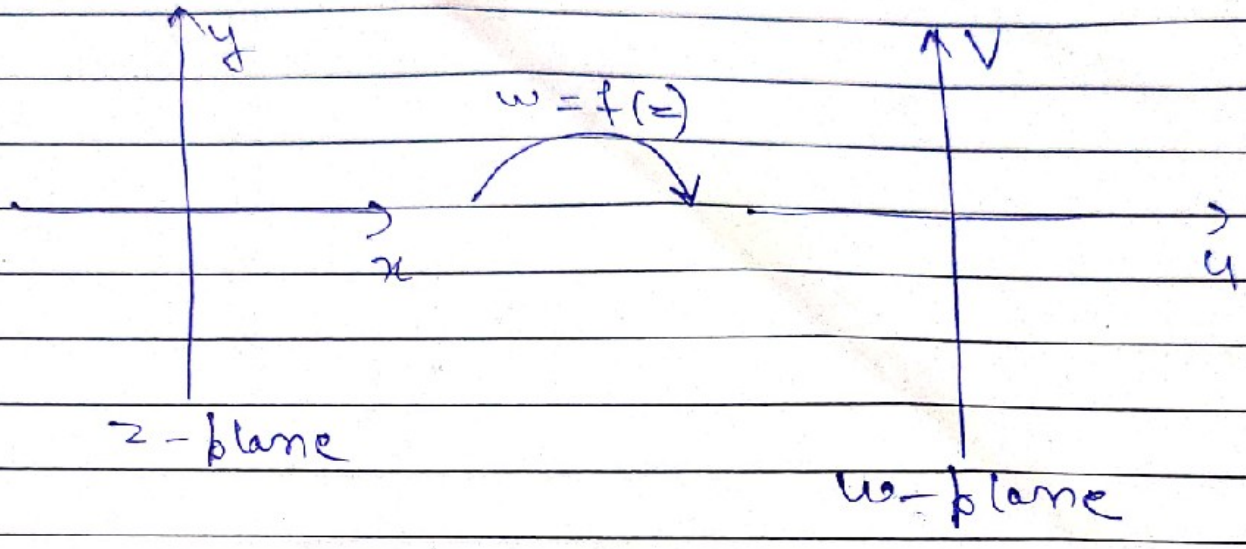
MAPPING



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BILINEAR TRANSFORMATION

$$w = \frac{az+b}{cz+d} \quad \text{---(1)}$$

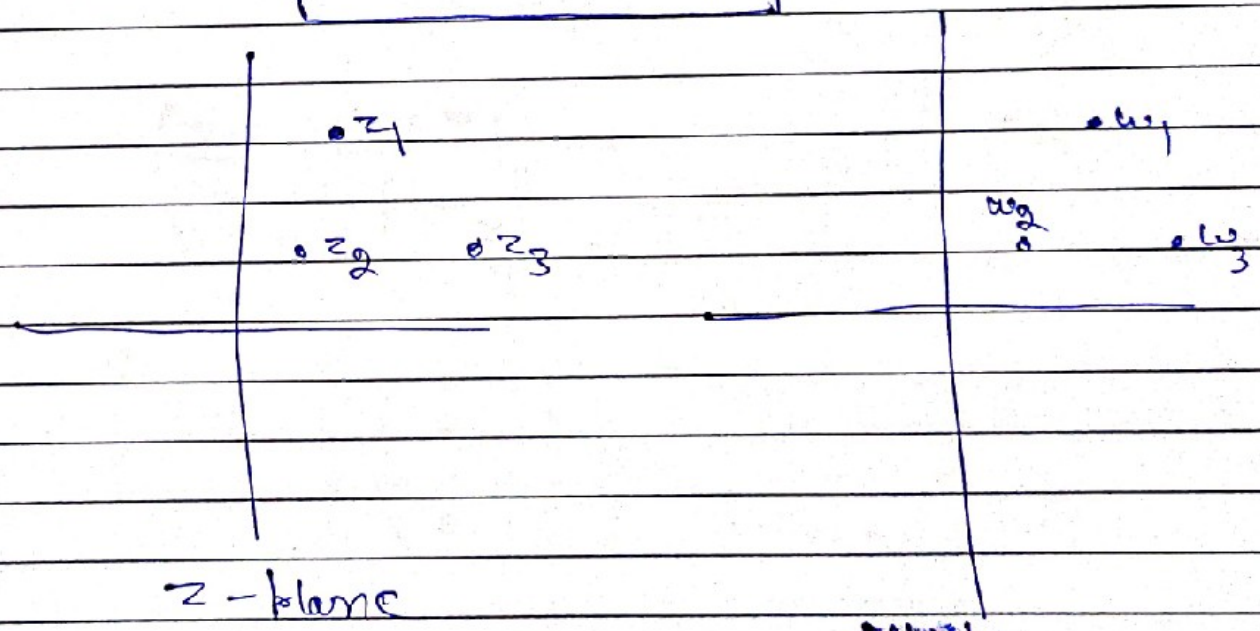
$$w(cz+d) = az+b$$

$$wcz + wd = az+b$$

$$wcz - az = b - wd$$

$$z(wc-a) = (b-wd)$$

$$z = \frac{b-wd}{wc-a}$$



$$\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}$$

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Q. Determine the Bilinear transformation which maps

$z_1 = 0, \quad z_2 = 1, \quad z_3 = \infty$

into $w_1 = i, \quad w_2 = -1, \quad w_3 = -i$

Soln:-

$$\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}$$

$$\left(\frac{z-z_1}{z_2-z_1}\right)\left(\frac{z_2-z_3}{z-z_3}\right) = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}$$

$$\left(\frac{z-z_1}{z_2-z_1}\right)\left(\frac{\frac{z_2}{z_3}-1}{\frac{z}{z_3}-1}\right) = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}$$

$$\left(\frac{z-0}{1-0}\right)\left(\frac{0-1}{0-1}\right) = \frac{(w-i)(-1+i)}{(w+i)(-1-i)}$$

$$\boxed{z = \frac{(w-i)(-1+i)}{(w+i)(-1-i)}}$$ Ans

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Q. Show that the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z|=1$ into the circle of radius unity in w plane and find the centre of the circle.

Soln:- Given

$$|z| = 1 \quad \text{---(i)}$$

$$w = \frac{5-4z}{4z-2}$$

$$w(4z-2) = 5-4z$$

$$4wz - 2w = 5 - 4z$$

$$4wz + 4z = 5 + 2w$$

$$z(4w+4) = (5+2w)$$

$$z = \frac{5+2w}{4w+4} \quad \text{---(2)}$$

Putting from (2) in (1)

$$\left|\frac{5+2w}{4w+4}\right| = 1$$

$$\frac{|5+2w|}{|4w+4|} = 1$$

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$$\frac{|5+2w|}{|4w+4|} = 1$$

$$|5+2w| = |4w+4|$$

$$|5+2(u+iv)| = |4(u+iv)+4|$$

$$|(5+2u)+i(2v)| = |(4u+4)+i(4v)|$$

$$\sqrt{(5+2u)^2+|2v|^2} = \sqrt{(4u+4)^2+(4v)^2}$$

Squaring both sides

$$(5+2u)^2+(2v)^2 = (4u+4)^2+(4v)^2$$

$$25+4u^2+20u+4v^2 = 16u^2+16+30u+16v^2$$

$$12u^2+12v^2+12u = 9$$

Dividing by 12 both sides

$$u^2+v^2+u = \frac{9}{12}^{3}$$

$$u^2+v^2+u = \frac{3}{4}$$

$$u^2+v^2+u+\left(\frac{1}{2}\right)^2 = \frac{3}{4}+\left(\frac{1}{2}\right)^2$$

$$\left(u+\frac{1}{2}\right)^2+(v)^2 = \frac{4}{4} = 1$$

$$\boxed{\left(u-\left(-\frac{1}{2}\right)\right)^2+(v-0)^2 = (1)^2}$$

Standard eqn of circle

$$(u-h)^2+(v-k)^2 = r^2$$

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∴ In w - plane the figure is circle of radius $r = 1$
centre $\left(-\frac{1}{2}, 0\right)$

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Q= Find the Image of the infinite strip

$$\frac{1}{4} \le y \le \frac{1}{2}$$

under the transformation $w = \frac{1}{z}$ also show the region geometrically.

Sol^{n} :-

$$w = \frac{1}{z}$$

$$w = \frac{1}{x+iy}$$

$$w = \frac{(x-iy)}{(x+iy)(x-iy)}$$

$$w = \frac{(x-iy)}{(x)^2-(iy)^2}$$

$$w = \frac{(x-iy)}{(x)^2+(y^2)}$$

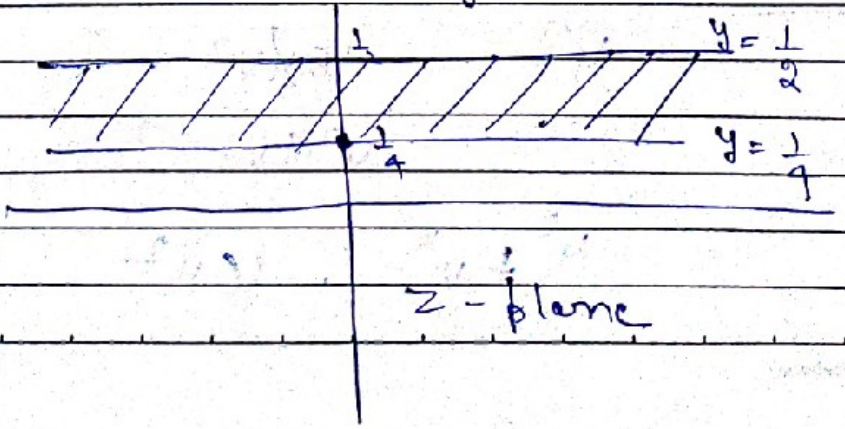
$$u+iv = w = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

$$u = \frac{x}{x^2+y^2}, \quad v = \frac{-y}{x^2+y^2}$$

$\rightarrow$ (1) $\rightarrow$ (2)

Ratio

$$\frac{u}{v} = \frac{-x}{y}$$



$y = \frac{1}{4}$

$\frac{y}{v} = -\frac{x}{(1/4)}$

$\frac{y}{v} = -4x$

$x = \frac{-y}{4v}$ ... (3)

from (1)

$y = \frac{\left(\frac{-y}{4v}\right)}{\left(\frac{-y}{4v}\right)^2 + \left(\frac{1}{4}\right)^2}$

$y = \frac{\frac{-y}{4v}}{\frac{y^2}{16v^2} + \frac{1}{16}}$

$y = \frac{\frac{-y}{4v}}{\frac{y^2 + v^2}{16v^2}}$

$y = -\frac{16yv^2}{y^2 + v^2}$

$y^2 + v^2 = -16v^2$

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$$u = \frac{\frac{-4}{4v}}{\frac{u^2+v^2}{16v^2}}$$

$$x = -\frac{x \cancel{16} v^2}{\cancel{4}v(u^2+v^2)}$$

$$u^2+v^2 = -4v$$

$$u^2+v^2+4v = 0$$

$$u^2+v^2+4v+(2)^2 = (2)^2$$

$$u^2+(v+2)^2 = (2)^2$$

$$(u-0)^2+(v-(-2))^2 = (2)^2$$

Circle centre $(0,-2)$
and
radius is $r = 2$

Proceeding similarly for $y = \frac{1}{2}$ we have

$$u^2+v^2+2v = 0$$

$$u^2+v^2+2v+1^2 = 1^2$$

$$u^2+(v+1)^2 = (1)^2$$

$$u^2+(v-(-1))^2 = (1)^2$$

$(0,-1)$ and $r = 1$

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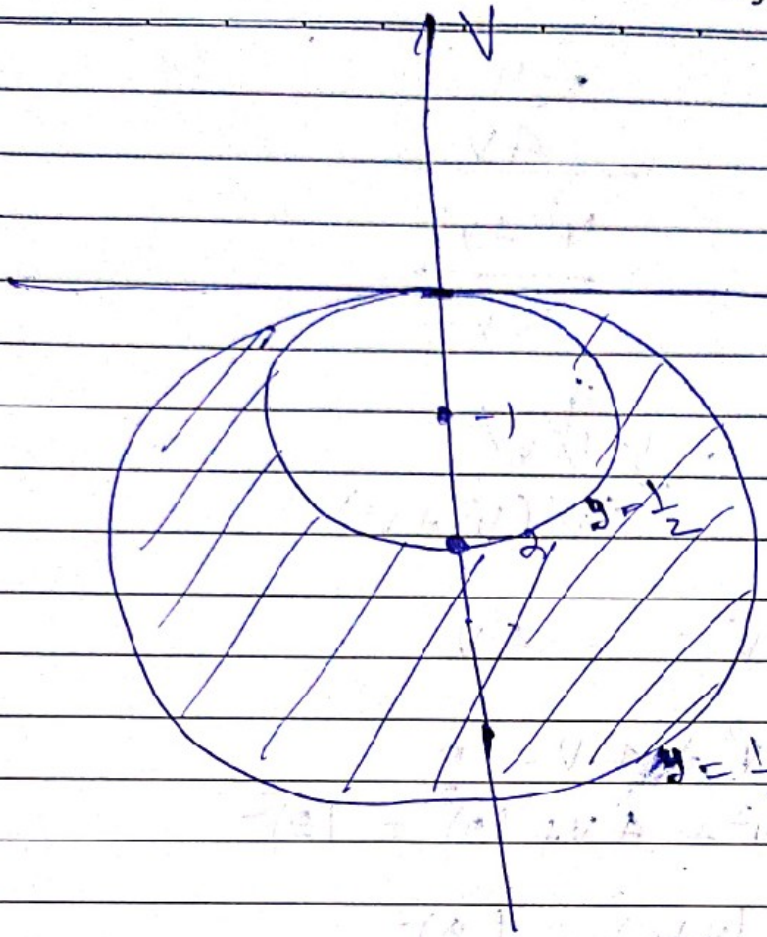

Ans